

Ph.D. Qualify Examination 2018
Theory of Computation

- This examination is closed books.
- Please turn off your cell phones.
- Remember that there are 2 pages of the qualify examination.
- Answer all questions as possible. You may have a partial score if you answer the correct direction.

1. Deterministic Finite Acceptor (DFA) (10 pts)

Find a dfa for the following language on $\Sigma = \{a, b\}$:

$$L = \{w : |w| \bmod 3 = 0, |w| \neq 6\}.$$

2. Nondeterministic Finite Acceptor (NFA) (10 pts)

Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

3. Find a regular grammar that generates the language on $\Sigma = \{a, b\}$ consisting of all strings with no more than two a 's. (10 pts)

4. Determine whether or not the following language on $\Sigma = \{a, b, c\}$ is regular: (10 pts)

$$L = \{a^n b^l a^k : k \leq n + l\}.$$

5. Show that the following grammar is ambiguous. (10 pts)

$$S \rightarrow aSbS | bSaS | \lambda.$$

6. Prove that all finite languages are regular. (10 pts)

7. Construct a nondeterministic pushdown automata that accepts the following language on $\Sigma = \{a, b, c\}$: (10 pts)

$$L = \{a^n b^{n+m} c^m : n \geq 0, m \geq 1\}.$$

8. Construct a dfa that accepts the language generated by the grammar: (10 pts)

$$S \rightarrow abA,$$

$$A \rightarrow baB,$$

$$B \rightarrow aA | bb.$$

9. Fill the following languages into the language hierarchy (If L_i is a regular language and also a context-free language, please fill L_i in the set of regular languages): (20 pts)

$$L_1 = \{a^n b^i c^j d^k : n + k \leq i + j\},$$

$$L_2 = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\},$$

$$L_3 = \{a^3 b^n c^n : n \geq 0\},$$

$$L_4 = \{a^n b^k c^n d^k : n \geq 0, k > n\},$$

$$L_5 = \{a^n b^n c^n : n \geq 0\},$$

$$L_6 = \{ww : w \in \{a, b\}^*\},$$

$$L_7 = \{ab, ad, ac, b, cd, acd\},$$

$$L_8 = \{a^{n!} : n \geq 0\},$$

$$L_9 = \{a^n : n \text{ is a prime number}\},$$

$$L_{10} = \{a^n b^j : n \leq j^2\}.$$

